

# Algebra

- 1) The perimeter of a rectangle is 12 units. The width of the rectangle is 9 units less than twice its length. What is the length of the rectangle? (1) 2004-WU2-1
- 2) What ordered pair of positive integers  $(m, n)$  satisfies  $7m + 12n = 43$ ? (1) 2004-WU2-9
- 3) Tyler's age is half of Mary's age. In four years, Tyler's age will be two-thirds Mary's age. How old is Tyler now? (2) 2004-WU3-9
- 4) Tim has three times as many coins as Mike. If Tim gives one coin to Mike, Mike will have a total number of coins equal to half the number of coins Tim started with. How many total coins do Tim and Mike have together? (2) 2004-WU6-3
- 5) The binary operation  $\#$  is defined as  $a \# b = 2a - 3b$ . What is the value of  $(4 \# 5) \# 6$ ? (1) 2004-WU8-3
- 6)  $x + 5 < 8$  and  $x$  is a prime number, what is the value of  $x$ ? (1) 2004-WU8-4
- 7) If  $x + y = 12$  and  $x - y = \frac{1}{12}$ , what is the value of  $x^2 - y^2$ ? (3) 2004-WU9-9
- 8) What is the coefficient of  $x^3$  when  $x^4 - 3x^3 + 5x^2 - 6x + 1$  is multiplied by  $2x^3 - 3x^2 + 4x + 7$  and the like terms combined? (3) 2004-WU10-4
- 9) Suppose  $5 \leq x \leq 8$ . What is the greatest possible positive difference of the expressions  $3x - 4$  and  $5 - 6x$ ? (1) 2004-WO5-10
- 10) Corn costs 99 cents per pound, and beans cost 45 cents per pound. If Sam buys 24 total pounds of corn and beans, and it costs \$18.09, how many pounds of corn did Sam buy? (3) 2004-WO7-5
- 11) Bryce bought 32 stools that required assembly. Some stools have three legs and other stools have four legs. The box arrived with 108 stool legs. If the four-legged stools cost \$20 and the three-legged stools cost \$15, how much did all of Bryce's stools cost? (3) 2004-WU15-4

12) A piece of wire 180 cm long is cut into two pieces with integer lengths. Each of the two pieces is formed into its own square with integer side lengths. The total area of the two squares is  $1073 \text{ cm}^2$ . How many centimeters longer is a side of the larger square than a side of the smaller square? (3) 2004-WU16-5

13) A right triangle has sides measuring  $2x - 1$ ,  $3x - 3$  and  $3x - 4$  units. What is the value of  $x$ ? (3) 2004-WO8-4

14) The area of a rectangular plot is 48,000 square meters. If Kim walks along the diagonal from one corner to the opposite corner instead of walking along the edges, she walks 120 meters less. What is the length of the diagonal? (3) 2004-WO8-9

15) Start with a positive integer; add 4; multiply by 2; subtract 3; multiply by 2; add 2; divide by 4; subtract 3. If the final result is 6, what was the value of the original integer? (2) 1999-WU2-6

16) What is the sum of the integer solutions to  $|x + 2| < 5$ ? (2) 1999-WU4-9

17) The chickens and pigs in Farmer McCoy's barn have a total of 50 heads and 170 legs. How many pigs are in the barn? (3) 1999-WU7-9

18) If  $a + b = 8$ ,  $b + c = -3$ , and  $a + c = -5$ , what is the value of the product  $abc$ ? (3) 1999-WU10-5

19) A rectangular pool measuring 6 feet by 12 feet is surrounded by a walkway. The width of the walkway is the same on all four sides of the pool. If the total area of the walkway and pool is  $520 \text{ ft}^2$ , what is the number of feet in the width of the walkway? (3) 1999-WU10-10

20) It takes Amelia 5 hours to mow the yard and it takes her brother Tom 7.5 hours to mow the same yard. If they have two lawn mowers, how many hours will it take for them to mow the yard together? (3) 1999-WU12-1

21) What is the greatest possible value of  $x + y$  such that  $x^2 + y^2 = 90$  and  $xy = 27$ ? (2) 1999-WU14-9

22) Which number can be subtracted from the numerator and added to the denominator of  $\frac{12}{15}$  to create a fraction that is equivalent to  $\frac{1}{2}$ ? (3) 1999-WU15-2

23) A fish tank weighs 80 pounds when 40% full of water, and it weighs 140 pounds when completely full. How many pounds does the tank weigh when empty? (3) 1999-WU15-9

24) Given that  $12a + 10b = 1020$ , what is the value of  $\frac{a}{5} + \frac{b}{6}$ ? (2) 1999-WU15-10

25) Rachel subtracted two positive numbers and the difference was 12. Bret multiplied the same two numbers and the product was 540. What is the sum of the numbers? (3) 1999-WU16-9

26) Given that  $f(x) = x^{-1} + \frac{x^{-1}}{1 + x^{-1}}$ , what is  $f(f(-2))$ ? 1999-WU17-7

27) The pressure  $P$  of wind exerted on a sail varies directly with the area  $A$  of the sail and the square of the velocity  $V$  of the wind; that is,  $P = kAV^2$ , where  $k$  is a constant. The pressure exerted on one square foot of sail is 1 pound when the velocity of the wind is 16 miles per hour. What is the velocity of the wind when the pressure on 9 square feet of sail is 49 pounds? Round to the nearest whole number? (2) 1999-WU18-5

28) For what value of  $x$  does  $1 + 2 + 3 + 4 + 5 + \dots + x = 120$ ? (3) 1999-WO3-4

29) If the hundreds and units digits of a three-digit number are switched, the new number is 693 greater than the original number. If the tens and units digits are switched, the new number is 27 greater than the original number. The sum of the digits is 14. What is the original number? (3) 1999-WO9-9

30) What number can be subtracted from both the numerator and denominator of  $\frac{19}{24}$  so that the resulting fraction will be equivalent to  $\frac{3}{4}$ ? (2) 2000-WU5-1

31) The mean weight of Jeb and Karl is 120 pounds. The mean weight of Karl and Doug is 140 pounds. The mean weight of Jeb and Doug is 135 pounds. Combined, how many pounds do Jeb, Karl and Doug weigh? (2) 2000-WU5-4

32) The function  $f$  is linear and satisfies  $f(d + 1) - f(d) = 3$  for all real numbers  $d$ . What is  $f(3) - f(5)$ ? (3) 2000-WU6-10

33) When several thieves tried to divide a sum of money by giving \$4 to each thief, one thief received nothing. When each thief took \$3, they had \$1 left over. What is the sum of the number of dollars and the number of thieves? (1) 2000-WU7-1

34) Simplify:  $[a - (b - c)] - [(a - b) - c]$  in terms of one variable. (2) 2000-WU9-6

35) Maria's dog weighs 30 pounds plus two-thirds of its own weight. How many pounds does Maria's dog weigh? (2) 2000-WU16-5

36) In the addition shown,  $a$ ,  $b$ ,  $c$  and  $d$  represent four different digits. What is the value of  $b$ ? (3) 2000-WU18-5

$$\begin{array}{r} a \\ + b \\ \hline c \end{array} \qquad \begin{array}{r} b \\ + c \\ \hline da \end{array}$$

37) If Ben gives me a penny, I will have twice as many pennies as he has. If I give Ben a penny, he and I will have the same number of pennies. How many pennies do I currently have? (2) 2000-WO6-6

38) When three integers are added two at a time, three distinct sums are obtained: 32, 48 and 46. What is the sum of the three integers? (2) 2000-WO8-10

39) Kevin's order of two hamburgers and three sodas costs \$4.13 at the Burger Spot. Alana orders four cheeseburgers and six sodas. Cheeseburgers cost 15 cents more than hamburgers. What is the cost of Alana's order? Round to the nearest cent. (3) 2003-WU4-6

40) What integer, added to both the numerator and the denominator of  $\frac{6}{7}$ , results in a fraction with a value of 0.8? (3) 2003-WU4-8

41) There are 12 ordered pairs of integers  $(x, y)$  that satisfy  $x^2 + y^2 = 25$ . What is the greatest possible sum of  $x + y$ ? (3) 2003-WU6-2

42) Megan has already saved \$18, and she earns \$4 per hour working four hours on each Saturday and three hours on each Sunday. Randy has already saved \$22, and she earns \$3 per hour and works six hours each week. If they combine their money, in how many weeks will they be able to afford a \$316 stereo? (2) 2003-WU6-5

43) A cliff at Worloe Bay is 36 feet above the water. If an object is dropped from the cliff, the formula  $h = 36 - 16t^2$  will give the height above the water's surface, in feet, of the object at  $t$  seconds from the time it was dropped. How many seconds does it take for an object dropped from the cliff to reach the water's surface? Round to the nearest tenth. (2) 2003-WO3-8

44) Children's tickets to a local play cost \$1.50 less than adult tickets. At one performance 325 children tickets and 175 adult tickets were sold for a total of \$3512.50. How many dollars did an adult ticket cost? (3) 2003-WO4-5

45) The formula  $T = 2\pi\sqrt{\frac{L}{32}}$  is used to find the time  $T$  in seconds for a pendulum of length  $L$  feet to complete one period (back and forth). In inches, how long must the pendulum be to have a period of 3.5 seconds? Round to nearest whole number. (3) 2003-WO5-10

46) In quadrilateral  $ABCD$ , the measure of angle  $A$  is  $10^\circ$  larger than the measure of angle  $C$ ; the measure of angle  $D$  is twice the measure of angle  $A$ ; and the measure of angle  $B$  is three times the measure of angle  $D$ . What is the measure of angle  $D$ ? (2) 2003-WU11-7

47) Solve for  $d$ :  $\frac{3}{2} = \frac{15}{c} = \frac{(c-d)}{4}$ . (2) 2003-WU12-3

48) What is the greatest integer that is a solution to the inequality

$$12 > \frac{x}{2} + 1? \quad (2) \quad 2003-WU12-9$$

49) When the sum of the digits of a number and the number itself are added, the result is 695. What is the number? (3) 2003-WU13-7

50) Given the proportions  $\frac{a}{b} = \frac{3}{7}$  and  $\frac{c}{b} = \frac{9}{14}$ , what is the value of  $\frac{c}{a}$ ? (2)

2003-WU14-6

51) For what value of  $x$  is the equation  $x + 2x + 3x + \dots + 100x = 100$  true? (3)

2003-WU13-7

52) Solve for  $x$ :  $\frac{x}{x+3} = 0.8125$ . (2) 2003-WU15-6

53) What non-zero integer must be placed in the square so that the simplified

product of these two binomials is a binomial:  $(3x + 2)(12x - \square)$ ? (3) 2003-WU16-1

54) A particular number's numerator is 14 less than its denominator. If 9 is added to both its numerator and its denominator the resulting value is  $\frac{1}{2}$ . What is the original number? (2) 2003-WO8-1

55) What is the solution to the equation  $|x - 7| = |x + 1|$ ? (2) 2003-WU13-7

56) A firefighter uses the formula  $S = 0.5n + 26$  to determine the maximum number of feet of horizontal distance ( $S$ ) water will travel leaving a hose with a  $\frac{3}{4}$ -inch nozzle diameter.  $N$  is the nozzle pressure in pounds. If the hose has a nozzle diameter greater than  $\frac{3}{4}$ -inch, 5 feet is added to the distance for every  $\frac{1}{8}$ -inch increased in nozzle diameter. For a  $1\frac{1}{8}$ -inch diameter nozzle with a nozzle pressure of 80 pounds, what is the maximum horizontal distance in feet that the water will travel? (3) 2003-WO8-9

57) An algebraic expression of the form  $a + bx$  has the value of 15 when  $x = 2$  and the value of 3 when  $x = 5$ . Calculate  $a + b$ . (2) 2002-WO1-4

58) What is the greatest integer solution to  $\pi x - 17 < 20$ ? (2) 2002-WO1-10

59) Let  $f(x) = 2x - 3$  and  $g(x) = x + 1$ . What is the value of  $f(1 + g(2))$ ? (3) 2002-WU3-5

60) If  $x + y = 12$ ,  $x + z = 14$  and  $y + z = 22$ , what is the product of  $x$ ,  $y$  and  $z$ ? (3) 2002-WU4-10

61) A quiz had only 3-point questions and 4-point questions. The best possible score is 100 and there are 29 questions. How many 4-point questions are there? (2) 2002-WO2-8

62) What is the sum of all integer values of  $x$  such that  $\frac{67}{2x - 23}$  is an integer? (2) 2002-WU7-6

63) Solve for  $n$ :  $6 = \frac{n}{1 + \frac{3}{1 + \frac{2}{1 + \frac{1}{1}}}}$  (2) 2002-WO4-9

64) The sum of two numbers is 15. One number is doubled and the other is tripled. The sum of the two new numbers is 39. What is the positive difference between the original numbers? (2) 2002-WU9-3

65) Cara has 162 coins in her collection of nickels, dimes and quarters, which has a total value of \$22. If Cara has twelve fewer nickels than quarters, how many dimes does she have? (3) 2002-WO5-6

66) Suppose  $a$ ,  $b$ ,  $c$  and  $d$  are integers satisfying:  $a - b + c = 3$ ,  $b - c + d = 4$ ,  $c - d + a = 1$  and  $d - a + b = 6$ . What is the value of  $a + b + c + d$ ? (3) 2002-WU11-10

67) The ratio of  $x$  to  $y$  is 1 to 2. What is the value of  $x$  if  $y = 4x - 36$ ? (3) 2002-WU12-3

68) Think of a number  $n$ . Double the number. Subtract 160. Divide the result by 4. Add 60. Subtract half the original number. Now square what you have. What is your answer? (3) 2002-WU16-6



69) If  $x + \frac{1}{x} = 6$ , then what is the value of  $x^2 + \frac{1}{x^2}$ ? (3) 2002-WO8-5

70) Suppose  $a * b$  is defined as  $2a - 3b$ . What is the value of  $(2 * 1) * (1 * 2)$ ? (2) 2002-WU18-7

71) The sum of the squares of three consecutive positive integers is 7805. What is the sum of the cubes of the three original integers? (3) 2002-WO9-4

72) What number can be subtracted from both the numerator and denominator of  $\frac{19}{24}$  so that the resulting fraction will be equivalent to  $\frac{3}{4}$ ? (2) 1997-WU9-9

73) What is the sum of the solutions to  $|4x - 6| = 4$ ? (3) 1997-WU10-10

74) If  $\frac{x-2}{y-1} = \frac{3}{5}$  and  $\frac{x-1}{y} = \frac{2}{3}$ , what is the product of  $x$  and  $y$ ? (3) 1997-WU16-5

75) If  $x^2 + 1 = 10$ , what is the least value of  $x$ ? (1) 1997-WU18-4

76) Kenneth is twice as old as Martin was when Kenneth was as old as Martin is now. Martin is now 18 years old. How old is Kenneth? (2) 1997-WO1-8

77) What is the sum of all integer solutions of  $x^4 - 25x^2 + 144 = 0$  (3) 1997-WO1-9

78) Mrs. Kiernan listed two positive numbers on the board and asked the students in her class to add the numbers. Tai subtracted the numbers by mistake and gave his answer as 37. Eileen multiplied them and gave her answer as 69,090. What was the correct sum? (3) 1997-WO2-4

79) A 24-year old woman has a 2-year old child. In how many years will the woman be twice as old as her child? (1) 1997-WO5-1

80) Suppose that  $a * b = a + b + ab$ . If  $x * 1 = 5$ , what is the value of  $x$ ? (1) 2001-WU1-8

81) Let  $x$  be a positive number and  $y$  be its reciprocal. Compute  $\frac{1}{x+1} + \frac{1}{y+1}$ .

(1) 2001-WU2-8

82) Given that  $4a + 5b + 7c = 13$  and  $4a + 3b + c = 19$ , what is the value of  $a + b + c$ ? (2) 2001-WU4-1

83) "A laborer was hired for a year, to be paid \$80 and a suit of clothes. After he worked 7 months, he left. Therefore, he earned  $\frac{7}{12}$  of his yearly salary. For his wages, he received the suit of clothes and \$35. What was the dollar value of the suit of clothes?" (3) 2001-WU5-9

84) Choose a number. Triple the number. Add 200. Double the result. Subtract 100. Divide by 4. Subtract 150% of the original number. What is the value of the result? (1) 2001-WU7-10

85) Solve for  $a$  in terms of  $b$  if two more than twice  $a$  is three less than the square of the number which is one less than  $b$ . (3) 2001-WU10-3

86) How many ordered pairs  $(a, b)$  of positive integers satisfy the equation  $3a + 4b < 12$ ? (1) 2001-WU10-6

87) When buying shirts at Sport's Shirts, there is a fixed set-up fee and a constant cost per shirt. The price for 20 baseball shirts would be \$390. An order for 80 baseball shirts would cost \$110. What would be the cost for 140 shirts? (1) 2001-WU11-2

88) The number  $y$  is 125% of another number  $x$ . What percent of  $8y$  is  $5x$ ? (3) 2001-WU12-3

89) Find an integer such that the sum of the integer and its square is 6 less than the square of the next greater integer. (3) 2001-WU18-2

90) James announces, "I just found \$5. I now have five times as much money than if I had lost \$5. How much money did James have before finding the \$5?" (2) 2001-WO2-7

91) Given:  $\frac{5}{13} = \frac{n}{39} = \frac{m+n}{156} = \frac{p-m}{104}$ . What is the value of  $p$ ? (2) 2001-WO6-3

92) What is the greatest integer value of  $x$  for which  $6x^2 + x - 2 < 0$ ? (3) 1996-WU11-1

93) Given that  $a$  and  $b$  are positive integers for which  $\frac{a}{4} + \frac{b}{5} = \frac{7}{10}$ , what is the sum of all the values of  $a$  and  $b$  that satisfy the equation? (3) 1996-WO2-3

94) Given that  $A$  is the set of all integral solutions of  $|x - 7| < 5$ , what is the median of all the members of set  $A$ ? (2) 1996-WO2-8

95) For a motor vehicle traveling at  $r$  miles per hour, the distance  $d$  in feet required to stop is approximately  $d = 0.05r^2 + r$ . If a school bus is traveling at 45 miles per hour, how many feet, to the nearest foot, would it travel before coming to a halt? (2) 1993-WU7-1

96) If half of a number, 25% of the number, and 0.1 times the number have a sum of 68, what is the number? (2) 1993-WU13-10

97) For  $0 < x < 1$ , which of the following is the largest? (2) 1993-WU15-9

$$\frac{1-x}{-1}, \quad x^2, \quad 1-x^2, \quad x^3, \quad \frac{1}{x}$$

98) Three rectangles have the same area. Their dimensions are:

1)  $x$  by  $y$

2)  $(x - 4)$  by  $(y + 3)$

3)  $(x + 2)$  by  $(y - 1)$

What is the area of each rectangle? (2) 1993-WU17-7

99) A student multiplied by four when they meant to add four. Then they added seven when they meant to multiply by seven. The answer they got was 39. What should they have gotten? (1) 1993-WU20-10

100) Under plan A, Carrie can be paid a monthly salary of \$600 plus a commission of 5% of total sales. Under plan B she will receive \$800 plus a commission of 6% on total sales in excess of \$10,000. What is the least amount of sales for which plan B would be equal to or better than plan A if sales are always more than \$10,000? (2) 1993-WO3-1

101) A rectangular lawn measures 60 feet by 80 feet. The edges of the lawn are mown up to install a sidewalk of uniform width around it. The area of the new lawn is one-sixth the area of the old. How many feet wide is the sidewalk? (2) 1993-WO3-2

102) Cindy purchased three blouses and two scarves for \$87.75. The two scarves were the same price, but one of the blouses cost \$5.00 more than the others. Five scarves would have cost \$64.75. What was the cost of the most expensive blouse? (2) 1993-WO4-1

103) Bottle A contains more soda than Bottle B. Pour from Bottle A into Bottle B as much soda as B already contains. Now pour from B into A as much soda as A now contains. Then pour from A into B as much soda as B now contains. Each bottle now contains 64 ounces. How many more ounces of soda were in Bottle A than in Bottle B at the beginning? (2) 1993-WU12-10

104) Find the mean of all solutions to  $|x - 2| = 2$ . (2) 1995-WU5-6

105) The table shown is a linear function. What is the value of  $w$  when  $h = 157$ ? (2) 1995-WU7-7

$w$	1	2	3
$h$	2	7	12

106) The sum of the squares of two integers is 290 and the square of their sum is 576. What is their product? (3) 1995-WU8-1

107) At a fishing tournament, you are awarded \$1 for each fish weighing 2 pounds or over, and you are assessed \$2 for each fish weighing less than 2 pounds. Given that Sue caught 12 fish, but neither made or lost money, how many of her fish weighed less than 2 pounds? (2) 1995-WU9-1

108) What is the largest integral value of  $x$  for which  $x^2 - 2x - 15 < 0$ ? (3) 1995-WU17-6

109) What is the largest integer  $n$  for which  $2^n > n!$ ? (1) 1995-WU17-8

110) What is the smallest composite number generated by  $p^2 - p - 1$  where  $p$  is a prime number? (1) *1995-WU20-4*

111) In a town,  $\frac{2}{3}$  of the adult men are married to  $\frac{3}{5}$  of the adult women. The number of married men and women are equal, and the adult population is over 100. What is the smallest possible number of residents in town? (3) *1995-WO3-3*

112) A three-digit number has 3 as its tens digit. Reversing the digits decreases the number by 297. The sum of the hundreds and units digits is 11. What is the number? (3) *1995-WO6-10*

113) Charlene's age in years is 16 more than the number of years in the sum of Betty's and Ashley's ages. The square of Charlene's age is 1632 more than the number of years in the square of the sum of Betty's and Ashley's ages. What is the sum of the ages of the three women? (3) *1995-WO8-6*

## *Algebra – Answer Key*

- |           |             |                |             |
|-----------|-------------|----------------|-------------|
| 1) 5      | 19) 7       | 38) 63         | 56) 81      |
| 2) (1, 3) | 20) 3       | 39) \$8.86     | 57) 19      |
| 3) 4      | 21) 12      | 40) -2         | 58) 11      |
| 4) 8      | 22) 3       | 41) 7          | 59) 5       |
| 5) -32    | 23) 40      | 42) 6          | 60) 240     |
| 6) 2      | 24) 17      | 43) 1.5        | 61) 13      |
| 7) 7/144  | 25) 48      | 44) \$8.00     | 62) 46      |
| 8) 19     | 26) $-8/3$  | 45) 119 in     | 63) 15      |
| 9) 63     | 27) 37      | 46) $74^\circ$ | 64) 3       |
| 10) 13.5  | 28) 15      | 47) 4          | 65) 70      |
| 11) \$540 | 29) 158     | 48) 21         | 66) 14      |
| 12) 11    | 30) 4       | 49) 676        | 67) 18      |
| 13) 11    | 31) 395 lbs | 50) $3/2$      | 68) 400     |
| 14) 14 m  | 32) -6      | 51) $2/101$    | 69) 34      |
| 15) 6     | 33) 21      | 52) 13         | 70) 14      |
| 16) -18   | 34) $2c$    | 53) 8          | 71) 398,259 |
| 17) 35    | 35) 90      | 54) $5/19$     | 72) 4       |
| 18) -120  | 36) 5       | 55) 3          |             |
|           | 37) 7       |                |             |

- |                              |               |          |
|------------------------------|---------------|----------|
| 73) 3                        | 91) 85        | 109) 3   |
| 74) 30                       | 92) 0         | 110) 155 |
| 75) -3                       | 93) 3         | 111) 114 |
| 76) 24                       | 94) 7         | 112) 734 |
| 77) 0                        | 95) 146 ft    | 113) 102 |
| 78) 527                      | 96) 80        |          |
| 79) 20                       | 97) $1/x$     |          |
| 80) 2                        | 98) 144       |          |
| 81) 1                        | 99) 28        |          |
| 82) 4                        | 100) \$40,000 |          |
| 83) 28                       | 101) 20 ft    |          |
| 84) 75                       | 102) \$23.95  |          |
| 85) $\frac{b^2 - 2b - 4}{2}$ | 103) 48 ozs   |          |
| 86) 3                        | 104) 2        |          |
| 87) \$1830                   | 105) 32       |          |
| 88) 50%                      | 106) 143      |          |
| 89) 5                        | 107) 4        |          |
| 90) \$7.50                   | 108) 4        |          |