Math 453 Name_____

Second Quarter Quarterly

Date_____

Solve each inequality.

1) $\dfrac{-3x-76}{2x+14} < -4$

2) $\dfrac{x^2-7x+10}{x+6} \geq 0$

3) $\dfrac{x-1}{x^2+9x+18} > 0$

4) $\dfrac{x+6}{x^2-14x+48} > 0$

5) $\dfrac{11}{x-5} > \dfrac{10}{x-4}$

6) $\dfrac{3}{2x-6} > \dfrac{1}{2x-4}$

7) $\dfrac{3x^2+22x-16}{3x-1} \leq 0$

8) $\dfrac{x^2-5x-14}{x-2} \leq 0$

9) $\dfrac{x^2+6x+9}{x-5} > 0$

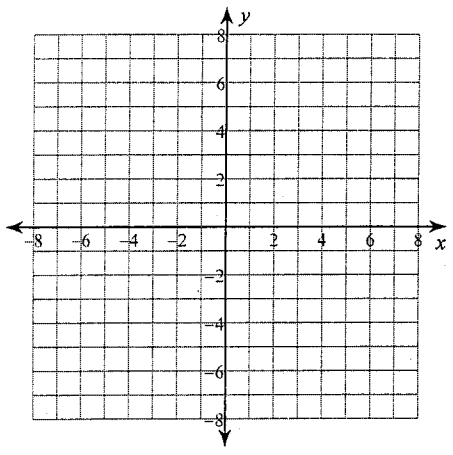
10) $\dfrac{x+8}{x^2-14x+48} < 0$

11) $\dfrac{x^2+x-30}{x^2-14x+49} > 0$

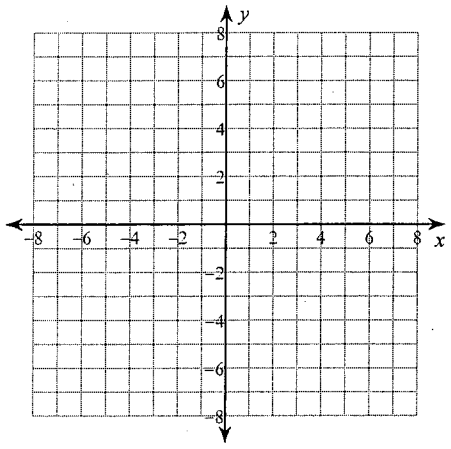
12) $\dfrac{x^2-16x+64}{x^2+x-12} > 0$

For each function, identify the holes, intercepts, horizontal asymptote, domain, limit behavior at all vertical asymptotes, and end behavior asymptote. Then sketch the graph.

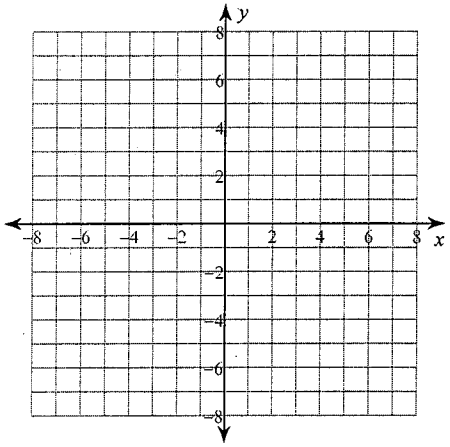
13) $f(x) = \dfrac{x+1}{2x+8}$



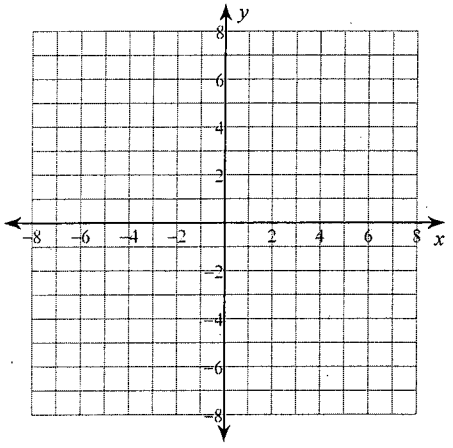
14) $f(x) = \dfrac{-x^2 - 4x}{x^2 + x - 6}$



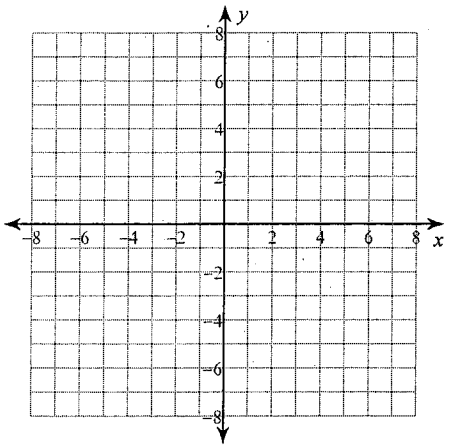
15) $f(x) = \dfrac{3x^2 + 9x - 12}{x^2 + x - 2}$



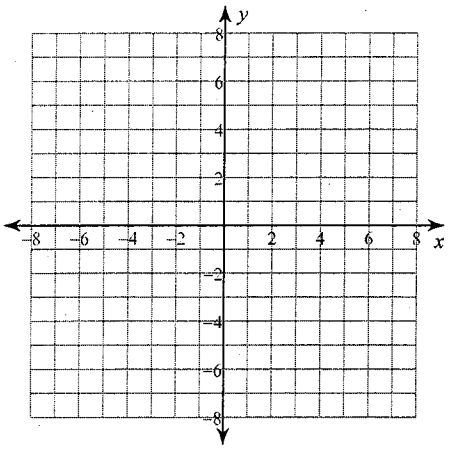
16) $f(x) = \dfrac{x^2 - 16}{-3x^2 + 9x}$



17) $f(x) = \dfrac{x-4}{-4x^3+4x^2+48x}$



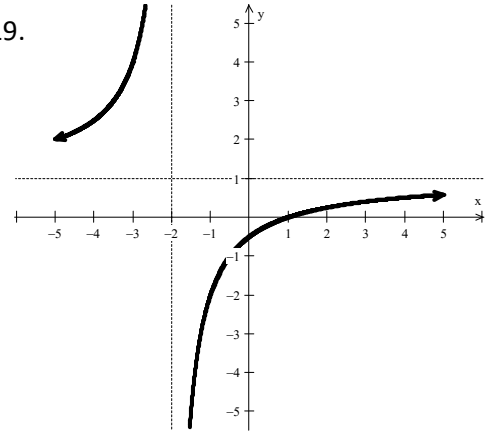
18) $f(x) = \dfrac{x^3-2x^2-8x}{-4x^3-20x^2-24x}$



Second Quarter Quarterly Review Cont.

For question 19-24, write the equation of the rational function based on the graph provided.

19.



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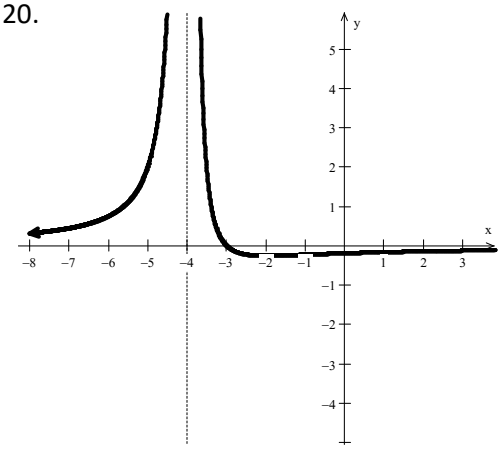

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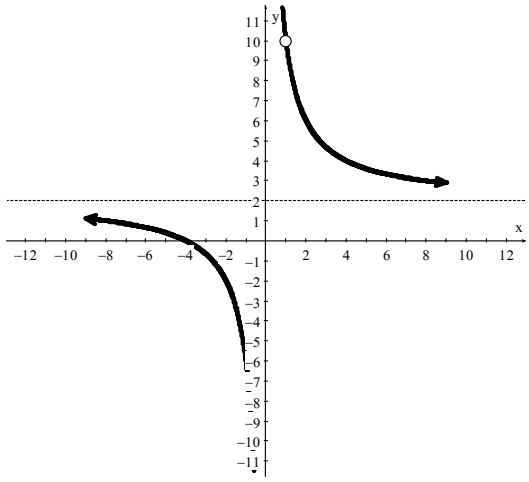

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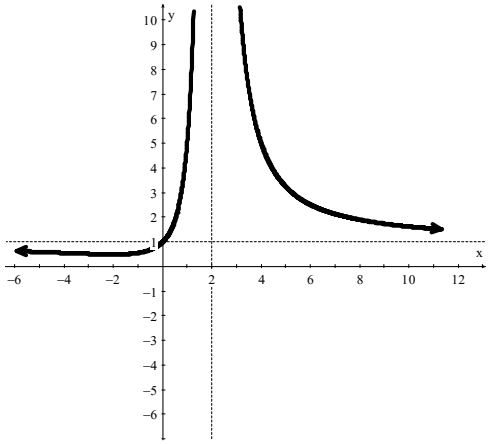

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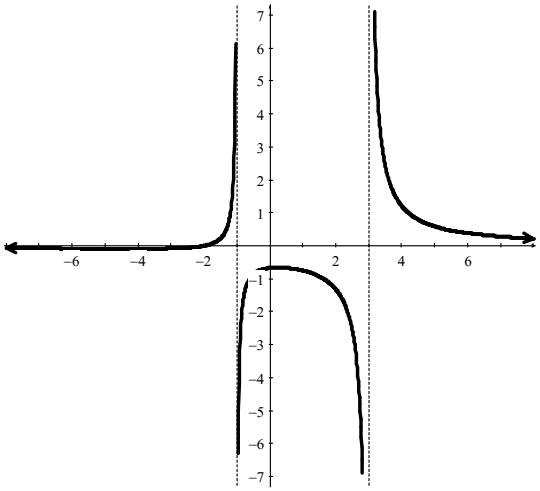

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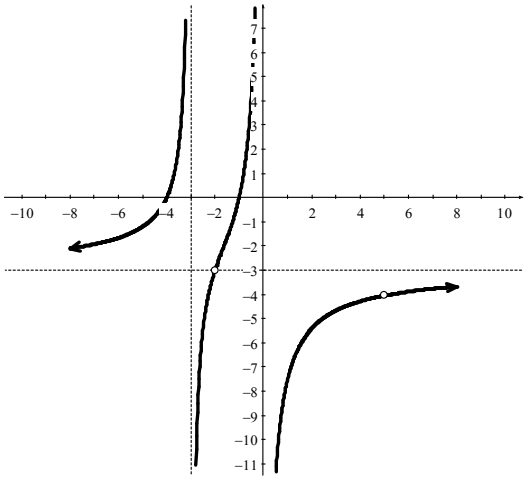
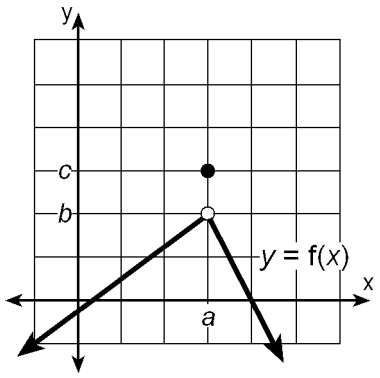


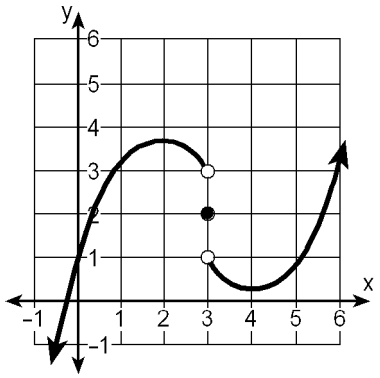
25) The graph of function f is shown below.



The $\lim_{x \to a} f(x)$ is

A) nonexistent
B) a
C) c
D) 0
E) b

26) The graph of function g is shown below.



Which of the following is *not* true?

A) $\lim_{x \to 3^-} f(x) = 3$
B) $\lim_{x \to 3} f(x)$ exists
C) $f(3) = 2$
D) $\lim_{x \to 1} f(x) = f(1)$
E) $\lim_{x \to 3^+} f(x) = 1$

27) $\lim_{x \to \infty} \frac{\sqrt{7x^2 - 2}}{x + 7} =$

28) $\lim_{x \to +\infty} \sqrt{x^2 + x + 1} =$

29) $\lim_{x \to -\infty} \frac{4x^5 - 3x^3 + 2}{12x^4 - 2x^2 + 3x - 1} =$

30) The $\lim_{x \to 5} x^2 - 3x + 6$ is

31) The $\lim_{x \to b} \frac{b - x}{x - b}$ is

32) The $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x^2 - 6x + 9}$ is

33) The $\lim_{x \to -1} \frac{x^2 + 6x + 5}{x^2 - 3x - 4}$ is

34) $\lim_{x \to 3} \frac{9 - x^2}{x^4 - 81}$ is

35) $\lim_{x \to 0} \frac{-5x^5 + 3x^3}{x}$ is

36) $\lim_{x \to 0^+} \frac{5x^3 + 8x^2}{3x^4 + 16x^3}$ is

37) $\lim_{h \to 0} \frac{(1 + h)^3 - 1}{h}$ is

38) $\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$ is

39) $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{x^2+x}\right)$ is

40) $\lim_{x\to 2}\frac{1-\frac{4}{x}+\frac{4}{x^2}}{1+\frac{1}{x}-\frac{6}{x^2}}$ is

41) $\lim_{x\to 7}\frac{x - 7}{\sqrt{x} - \sqrt{7}}$ is

42) Given function f defined below.

$$f(x)=\begin{cases} x^2+2x\ , & x\le a \\ x+2\ , & x>a \end{cases}$$

Determine all values of a for which $\lim_{x\to a} f(x)$ exists.

43) Let $f(x)=\begin{cases} \frac{x-3}{|x-3|}\ , & x\neq 3 \\ 3\ , & x=3 \end{cases}$.

Which of the following statements are true?

I. $\lim_{x\to 3^+} f(x) = 1$

II. $\lim_{x\to 3^-} f(x) = -1$

III. $\lim_{x\to 3} f(x) = 3$

A) *I*, only
B) *II*, only
C) *I* and *II*, only
D) *I*, *II*, and *III*
E) None of the statements are true.

Questions 44 through 35 refer to the following:

For the given function, find the limit, if it exists.

44) $\lim_{t\to 0}\sqrt{(3 - t)^4}$

45) $\lim_{x\to -\infty} 4x^5 - 3x^3 + x - 7$

46) $\lim_{x\to -\infty} 10x^6 + 5x^5 - 2x$

47) $\lim_{x\to -\infty}\frac{7 - x}{x^2 - 49}$

48) $\lim_{x\to \infty}\frac{x^3 + 4x - 3}{x^5 - 4x^2 + 2x}$

49) $\lim_{x\to \infty}\frac{2x^4 + 6x}{(x^2 - 1)(x^2 + 1)}$

50) $\lim_{x\to -\infty}\frac{\sqrt{3x^2 - 4}}{x + 4}$

51) $\lim_{x\to 9}\frac{x^2 - 81}{3 - \sqrt{x}}$

52) $\lim_{x\to 0} x\sqrt{x - 1}$

53) $\lim_{x\to 2^-}\frac{x + 2}{(x - 2)^3}$

54) $\lim_{x\to 0^+}\frac{x}{|x| + x}$

55) $\lim_{x \to 3^+} \dfrac{3 - x}{|x - 3|}$

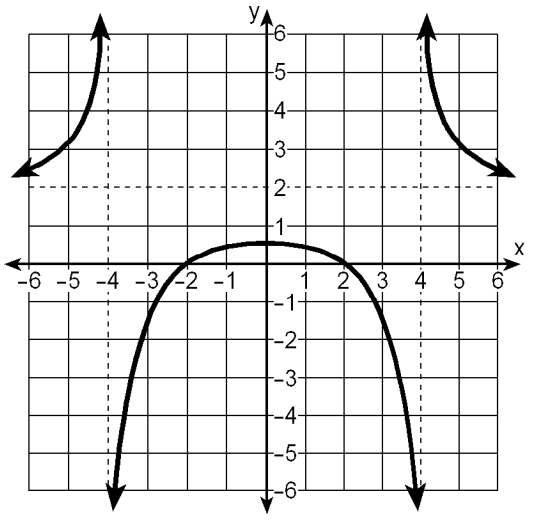
56) $\lim_{x \to -3} \dfrac{x + 3}{x^2 + 6x + 9}$

57) $\lim_{x \to 3} \dfrac{x^2}{x^2 - 9}$

58) $\lim_{x \to 5} \dfrac{\frac{1}{5} - \frac{1}{x}}{5 - x}$

59) $\lim_{x \to -4} 4\pi^7$

60) Which of the following equations *best* represents the accompanying graph?



A) $f(x) = \dfrac{4}{x^2 - 16}$

B) $f(x) = \dfrac{8 - 2x^2}{x^2 - 16}$

C) $f(x) = \dfrac{x^2 + 4}{x^2 - 16}$

D) $f(x) = \dfrac{2x^2 - 8}{x^2 - 16}$

E) $f(x) = \dfrac{x^2 - 4}{x^2 - 16}$

61) An asymptote for $g(x) = \dfrac{(x + 3)(x - 2)}{x + 5}$ is

A) $y = -5$

B) $x = -5$

C) $x = 2$

D) $x = -3$

E) $y = 5$

62) The graph of which of the following functions has $y = -2$ as a horizontal asymptote?

A) $f(x) = \dfrac{|x - 2|}{x + 2}$

B) $f(x) = \dfrac{x^2}{x^2 - 4}$

C) $f(x) = \dfrac{2x^2}{4 - x^2}$

D) $f(x) = \dfrac{x^2 - 4}{2x^2}$

E) $f(x) = \dfrac{2x^2}{4 + x^2}$

63) The graph of which function has $y = 1$ as an asymptote?

A) $f(x) = \dfrac{1}{\frac{1}{x} - 1}$

B) $f(x) = \dfrac{-x}{\frac{1}{x} - 1}$

C) $f(x) = \dfrac{-1}{\frac{1}{x} - 1}$

D) $f(x) = \dfrac{x}{1 - x}$

E) $f(x) = \dfrac{-1}{\frac{1}{x} + 1}$

64) Which of the following is *not* an equation of an asymptote for the graph of $f(x) = \dfrac{2x}{1 - |x|}$?

A) $x = 1$

B) $y = 2$

C) $y = -2$

D) $x = -1$

E) $y = 1$

65) Which of the following statements is true for the graph of $f(x) = \frac{x^2 + x + 4}{2x^2 - x}$?

A) $y = 0$ is a horizontal asymptote and $x = 0$ and $x = \frac{1}{2}$ are vertical asymptotes.

B) Function f has no horizontal asymptotes, but $x = 0$ and $x = \frac{1}{2}$ are vertical asymptotes.

C) $y = \frac{1}{2}$ is a horizontal asymptote and $x = \frac{1}{2}$ is the only vertical asymptote.

D) $y = \frac{1}{2}$ is a horizontal asymptote and $x = 0$ and $x = \frac{1}{2}$ are vertical asymptotes.

E) $y = 0$ is a horizontal asymptote, but function f has no vertical asymptotes.

66) Which of the following states the equations of all asymptotes, both vertical and horizontal, for the graph of $f(x) = \frac{2x^3 + 4x}{x + 2x^2 - 3x^3}$?

A) $x = -\frac{1}{3}$ and $x = 1$ are vertical asymptotes and $y = -\frac{2}{3}$ is a horizontal asymptote.

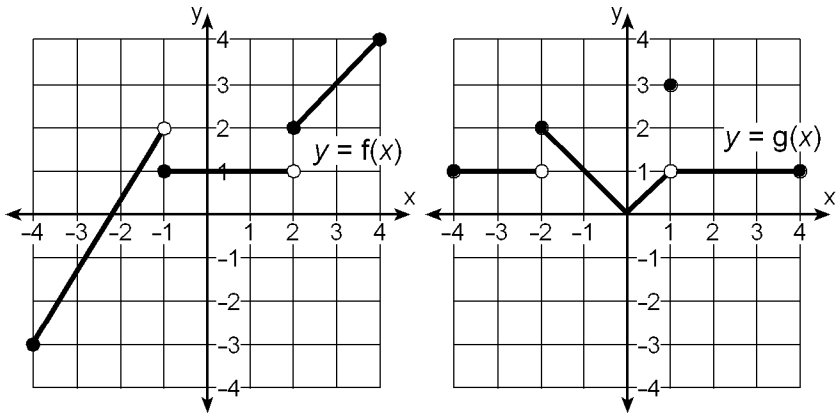
B) $x = -\frac{1}{3}$, $x = 0$, and $x = 1$ are vertical asymptotes and $y = -\frac{2}{3}$ is a horizontal asymptote.

C) $x = -\frac{1}{3}$ and $x = 1$ are vertical asymptotes and $y = 0$ is a horizontal asymptote.

D) $x = -\frac{1}{3}$, $x = 0$, and $x = 1$ are vertical asymptotes and $y = \frac{2}{3}$ is a horizontal asymptote.

E) $x = -\frac{1}{3}$ and $x = 1$ are vertical asymptotes and $y = \frac{2}{3}$ is a horizontal asymptote.

67) Shown below are the graphs of two functions, f and g.



Which of the following limits exist for these functions?

I. $\lim_{x \to 1} [f(x) + g(x)]$

II. $\lim_{x \to -2} [f(x) \cdot g(x)]$

III. $\lim_{x \to 0} \left(\frac{f(x)}{g(x)} \right)$

A) *I* and *II*, only
B) *I*, *II*, and *III*
C) *II*, only
D) *I* and *III*, only
E) *I*, only

Answers to Second Quarter Quarterly

1) $(-7, 4)$

2) $(-6, 2] \cup [5, \infty)$

3) $(-6, -3) \cup (1, \infty)$

4) $(-6, 6) \cup (8, \infty)$

5) $(-6, 4) \cup (5, \infty)$

6) $\left(\frac{3}{2}, 2\right) \cup (3, \infty)$

7) $(-\infty, -8] \cup \left(\frac{1}{3}, \frac{2}{3}\right]$

8) $(-\infty, -2] \cup (2, 7]$

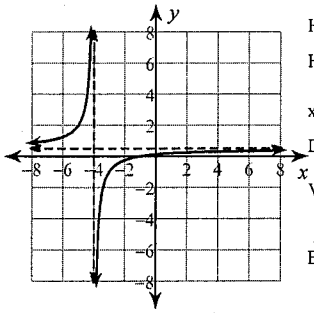
9) $(5, \infty)$

10) $(-\infty, -8) \cup (6, 8)$

11) $(-\infty, -6) \cup (5, 7) \cup (7, \infty)$

12) $(-\infty, -4) \cup (3, 8) \cup (8, \infty)$

13)



Holes: None

Horz. Asym.: $y = \frac{1}{2}$

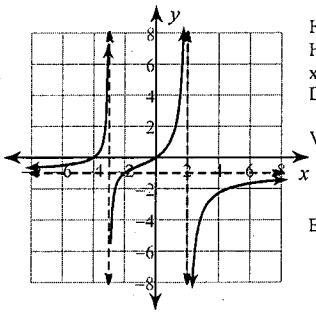
x-intercepts: -1, y-intercept: $\frac{1}{8}$

Domain:
All reals except -4

Vert. Asym. behavior:
$\lim_{x \to -4^-} f(x) = \infty$, $\lim_{x \to -4^+} f(x) = -\infty$

End behavior asym.: $y = \frac{1}{2}$

14)



Holes: None

Horz. Asym.: $y = -1$

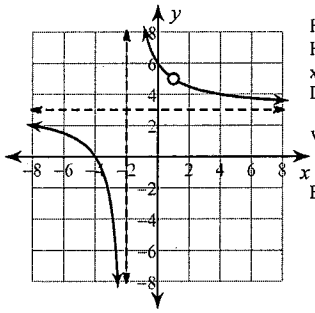
x-intercepts: 0, -4, y-intercept: 0

Domain:
All reals except -3, 2

Vert. Asym. behavior:
$\lim_{x \to -3^-} f(x) = \infty$, $\lim_{x \to -3^+} f(x) = -\infty$
$\lim_{x \to 2^-} f(x) = \infty$, $\lim_{x \to 2^+} f(x) = -\infty$

End behavior asym.: $y = -1$

15)



Holes: $x = 1$

Horz. Asym.: $y = 3$

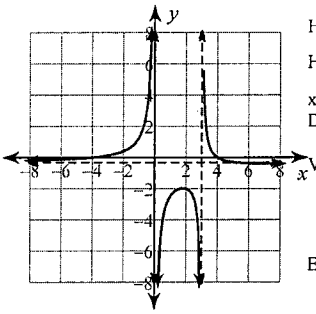
x-intercepts: -4, y-intercept: 6

Domain:
All reals except -2, 1

Vert. Asym. behavior:
$\lim_{x \to -2^-} f(x) = -\infty$, $\lim_{x \to -2^+} f(x) = \infty$

End behavior asym.: $y = 3$

16)



Holes: None

Horz. Asym.: $y = -\frac{1}{3}$

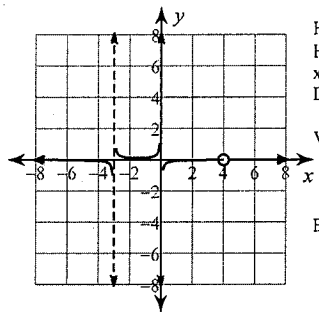
x-intercepts: 4, -4, y-intercept: None

Domain:
All reals except 0, 3

Vert. Asym. behavior:
$\lim_{x \to 0^-} f(x) = \infty$, $\lim_{x \to 0^+} f(x) = -\infty$
$\lim_{x \to 3^-} f(x) = -\infty$, $\lim_{x \to 3^+} f(x) = \infty$

End behavior asym.: $y = -\frac{1}{3}$

17)



Holes: $x = 4$

Horz. Asym.: $y = 0$

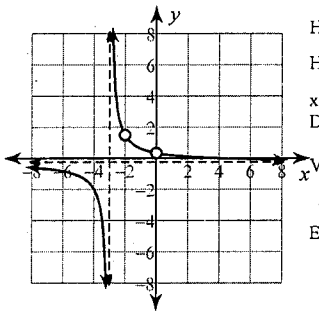
x-intercepts: None, y-intercept: None

Domain:
All reals except -3, 0, 4

Vert. Asym. behavior:
$\lim_{x \to -3^-} f(x) = -\infty$, $\lim_{x \to -3^+} f(x) = -\infty$
$\lim_{x \to 0^-} f(x) = -\infty$, $\lim_{x \to 0^+} f(x) = -\infty$

End behavior asym.: $y = 0$

18)



Holes: $x = -2$, $x = 0$

Horz. Asym.: $y = -\frac{1}{4}$

x-intercepts: 4, y-intercept: None

Domain:
All reals except -3, -2, 0

Vert. Asym. behavior:
$\lim_{x \to -3^-} f(x) = -\infty$, $\lim_{x \to -3^+} f(x) = \infty$

End behavior asym.: $y = -\frac{1}{4}$

Key to 19- 24

19. $f(x)=\dfrac{x-1}{x+2}$

20. $f(x)=\dfrac{x+3}{(x+4)^2}$

21. $f(x)=\dfrac{2(x+4)(x-1)}{x(x-1)}$

22. $f(x)=\dfrac{x^2+4}{(x-2)^2}$

23. $f(x)=\dfrac{x+2}{(x-3)(x+1)}$

24. $f(x)=\dfrac{-3(x+1)(x+4)(x+2)(x-5)}{x(x+3)(x+2)(x-5)}$

25) E 26) B

27) $\sqrt{7}$

28) $+\infty$

29) $-\infty$

30) 16

31) -1

32) nonexistent

33) $-\frac{4}{5}$

34) $-\frac{1}{18}$

35) 0

36) $+\infty$

37) 3

38) nonexistent

39) 1

40) 0

41) $2\sqrt{7}$

42) $a = -2$ and $a = 1$

43) C

44) 9

45) $-\infty$

46) $+\infty$

47) 0

48) 0

49) 2

50) $-\sqrt{3}$

51) -108

52) nonexistent

53) $-\infty$

54) $\frac{1}{2}$

55) -1

56) nonexistent

57) nonexistent

58) $-\frac{1}{25}$

59) $4\pi^7$

60) D 61) B 62) C 63) C 64) E

65) D 66) A 67) E