$$f(x) = \begin{cases} x+5 & x<1 \\ x+7 & x \geq 1 \end{cases}$$

$$\lim_{x \to 1^-} f(x) = 6 \qquad \lim_{x \to 1^+} f(x) = 8$$

6.

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	0.0408	0.0401	0.0400	0.0400	0.0399	0.0392

$$\lim_{x\to 4}\frac{[x/(x+1)]-(4/5)}{x-4}\approx 0.04 \qquad \left(\text{Actual limit is } \tfrac{1}{25}.\right)$$

9. $\lim_{x\to 2} f(x) = \lim_{x\to 2}(4-x) = 2$

27. (a) $\lim_{x\to c}[5g(x)] = 5\lim_{x\to c} g(x) = 5(3) = 15$

(b) $\lim_{x\to c}[f(x)+g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = 2+3 = 5$

(c) $\lim_{x\to c}[f(x)g(x)] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right] = (2)(3) = 6$

(d) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{2}{3}$

42. $$\lim_{x\to 4}\frac{x^2-5x+4}{x^2-2x-8} = \lim_{x\to 4}\frac{(x-4)(x-1)}{(x-4)(x+2)} = \lim_{x\to 4}\frac{(x-1)}{(x+2)} = \frac{3}{6} = \frac{1}{2}$$

43. $$\lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x} = \lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x}\cdot\frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}$$
$$= \lim_{x\to 0}\frac{(x+5)-5}{x\left(\sqrt{x+5}+\sqrt{5}\right)} = \lim_{x\to 0}\frac{1}{\sqrt{x+5}+\sqrt{5}}$$
$$= \frac{1}{2\sqrt{5}} = \frac{\sqrt{5}}{10}$$

48. $$\lim_{x\to 0}\frac{\frac{1}{x+4}-\frac{1}{4}}{x} = \lim_{x\to 0}\frac{\frac{4-(x+4)}{4(x+4)}}{x} = \lim_{x\to 0}\frac{-1}{4(x+4)} = -\frac{1}{16}$$

8. $\lim_{x\to 2^+}\frac{2-x}{x^2-4} = \lim_{x\to 2^+} -\frac{1}{x+2} = -\frac{1}{4}$

9. $\lim_{x\to -3^-}\frac{x}{\sqrt{x^2-9}} = -\infty$

11. $\lim_{x\to 0^-}\frac{|x|}{x} = \lim_{x\to 0^-}\frac{-x}{x} = -1$

15. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\frac{x+2}{2} = \frac{5}{2}$

29. $\lim_{x\to 2^+}\frac{x-3}{x-2} = -\infty$

32. $\lim_{x\to 4^-}\frac{x^2}{x^2+16} = \frac{1}{2}$

35. $\lim_{x\to 1}\frac{x^2-x}{(x^2+1)(x-1)} = \lim_{x\to 1}\frac{x}{x^2+1} = \frac{1}{2}$

Home work

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p 234 12, 17

p 243 34, 38, 42, 43

Home work

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p 234 18, 27, 29, 41, 49

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23. (a) $\lim_{x\to 1} f(x) = 5 - 1 = 4$

(b) $\lim_{x\to 4} g(x) = 4^3 = 64$

(c) $\lim_{x\to 1} g(f(x)) = g(f(1)) = g(4) = 64$

26. (a) $\lim_{x\to 4} f(x) = 2(4^2) - 3(4) + 1 = 21$

(b) $\lim_{x\to 21} g(x) = \sqrt[3]{21 + 6} = 3$

(c) $\lim_{x\to 4} g(f(x)) = g(21) = 3$

47. $$\lim_{x\to 0} \frac{\dfrac{1}{3+x} - \dfrac{1}{3}}{x} = \lim_{x\to 0} \frac{3 - (3+x)}{3(3+x)x}$$
$$= \lim_{x\to 0} \frac{-1}{3(3+x)} = \frac{-1}{9}$$

57. $f(x) = 2x + 3$

$$\lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x\to 0} \frac{2(x+\Delta x) + 3 - (2x+3)}{\Delta x}$$
$$= \lim_{\Delta x\to 0} \frac{2x + 2\,\Delta x - 2x}{\Delta x}$$
$$= \lim_{\Delta x\to 0} \frac{2\,\Delta x}{\Delta x} = \lim_{\Delta x\to 0} 2 = 2$$

12. $$\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} \frac{x-2}{x-2} = 1$$

17. $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x+1) = 2$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^3+1) = 2$

$\lim_{x\to 1} f(x) = 2$

34. $$\lim_{x\to -(1/2)^+} \frac{6x^2 + x - 1}{4x^2 - 4x - 3}$$
$$= \lim_{x\to -(1/2)^+} \frac{3x - 1}{2x - 3} = \frac{5}{8}$$

38. $$\lim_{x\to 0^-} \left(x^2 - \frac{1}{x}\right) = \infty$$

42. $$\lim_{x\to 4} [f(x)g(x)] = \lim_{x\to 4} \frac{x^2 - 5x}{(x-4)^2} = -\infty$$

43. $$\lim_{x\to 4} \left[\frac{f(x)}{g(x)}\right]$$
$$= \lim_{x\to 4} \frac{1}{(x-4)^2(x^2 - 5x)} = -\infty$$

57. $f(x) = 2x + 3$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$f(\boxed{x+\Delta x})$

$f(2)$

$f(a)$

$f(\sqrt{8})$

$$\lim_{\Delta x \to 0} \frac{[2(x+\Delta x)+3] - [2x+3]}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2x+2\Delta x+3-2x-3}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 2 = \boxed{2}$$

26 c) $f(x) = 2x^2 - 3x + 1$ $g(x) = \sqrt[3]{x+6}$

$f(4) = 2(16) - 3(4) + 1$

$$\lim_{x \to 4} g(f(x))$$

$$g\left(\lim_{x \to 4} f(x)\right)$$

$$g(21) = \sqrt[3]{21+6} = \sqrt[3]{27} = 3$$

58. $\lim_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \dfrac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$

$= \lim_{\Delta x \to 0} \dfrac{x + \Delta x - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$

$= \lim_{\Delta x \to 0} \dfrac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \dfrac{1}{2\sqrt{x}}$

Home work

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18. $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (1 - x) = 0$

27. $g(x) = \sqrt{25 - x^2}$ is continuous on $[-5, 5]$.

29. $\lim_{x \to 0^-} f(x) = 3 = \lim_{x \to 0^+} f(x)$.

f is continuous on $[-1, 4]$.

41. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

has a **possible** discontinuity at $x = 1$.

1. $f(1) = 1$

2. $\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1 \\ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 = 1 \end{array}\right\} \lim_{x \to 1} f(x) = 1$

3. $f(1) = \lim_{x \to 1} f(x)$

f is continuous at $x = 1$, therefore, f is continuous for all real x.

44. $\lim_{x \to 4} \left[\dfrac{g(x)}{f(x)}\right] = \lim_{x \to 4} (x^2 - 5x)(x - 4)^2 = 0$

49. Find a such that

$\lim_{x \to 2^+} ax^2 = 4a$

$= \lim_{x \to 2^-} x^3 = 8.$

$4a = 8$

$a = 2$

$f(x) = \sqrt{x}$

$f(x + \Delta x)$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{x} + \Delta x - \cancel{x}}{\Delta x\left[\sqrt{x+\Delta x} + \sqrt{x}\right]}$$

$$\lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Home work

Handout

59. $f(x) = \dfrac{4}{x}$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{4}{x + \Delta x} - \dfrac{4}{x}}{\Delta x}$$

$x(x+\Delta x)$

$x(x+\Delta x)$

$$= \lim_{\Delta x \to 0} \frac{4x - 4(x + \Delta x)}{(x + \Delta x)x\,\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-4\cancel{\Delta x}}{(x + \Delta x)x\cancel{\Delta x}} = \lim_{\Delta x \to 0} \frac{-4}{(x + \Delta x)x} = \frac{-4}{x^2}$$

$\dfrac{-4}{x \cdot x}$

26. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$ has discontinuity at $x = 1$ since $f(1) = 2 \neq \lim_{x \to 1} f(x) = 1$.

33. $f(x) = \dfrac{x}{x^2 - x}$ is not continuous at $x = 0, 1$.

$x = 0$ is a removable $\quad x = 1$ is a nonremovable

38. $f(x) = \dfrac{x - 1}{(x + 2)(x - 1)}$

nonremovable discontinuity at $x = -2$

removable discontinuity at $x = 1$

42. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$

has a **possible** discontinuity at $x = 1$.

1. $f(1) = 1^2 = 1$

2. $\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (-2x + 3) = 1 \\ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 = 1 \end{array}\right\} \lim_{x \to 1} f(x) = 1$

3. $f(1) = \lim_{x \to 1} f(x)$

f is continuous at $x = 1$, therefore, f is continuous for all real x.