Math 453 Name________________________

Home work Limits and Discontinuities Date______________

Evaluate each limit.

1) $\lim_{h \to 0} \frac{\sqrt{5+h} - \sqrt{5}}{h}$

2) $\lim_{h \to 0} \frac{\left(\frac{1}{2}+h\right)^2 - \left(\frac{1}{2}\right)^2}{h}$

$\left(\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right) = \frac{1}{4} + \frac{1}{2}h + \frac{1}{2}h + h^2$

$\frac{1}{4} + 1h + h^2$

$\lim_{h \to 0} \frac{\cancel{\frac{1}{4}} + 1h + h^2 - \cancel{\frac{1}{4}}}{h}$

$\lim_{h \to 0} \frac{h + h^2}{h} = \lim_{h \to 0} 1 + h = \boxed{1}$

Determine if each function is continuous. If the function is not continuous, find the x-axis location of and classify each discontinuity.

3) $f(x) = \frac{x-5}{x^2 - 2x - 3}$

4) $f(x) = \begin{cases} \frac{x}{2} + 4, & x \le 0 \\ x^2 - 4x + 4, & x > 0 \end{cases}$

$\lim_{x \to 0^-} \frac{x}{2} + 4 = 4$

$\lim_{x \to 0^+} x^2 - 4x + 4 = 4$

$f(0) = 4$

5) $f(x) = -\frac{x-1}{x^2 - 3x + 2}$

6) $f(x) = \begin{cases} 2 - \frac{x}{2}, & x \ne -3 \\ 1, & x = -3 \end{cases}$

$\lim_{x \to 3^-} 2 - \frac{x}{2}$

$= 2 - \frac{3}{2}$

$\lim_{x \to 3^+} 2 - \frac{x}{2}$

$2 - \frac{3}{2}$

$f(-3) = 1$

Answers to Home work Limits and Discontinuities

1) $\frac{\sqrt{5}}{10}$

2) 1

3) Infinite discontinuities at: $x = -1$, $x = 3$

4) Continuous

5) Removable discontinuity at: $x = 1$
Infinite discontinuity at: $x = 2$

6) Removable discontinuity at: $x = -3$

Home work

p 235 36, 39, 50

p 260 6, 8, 9

36. $f(x) = \dfrac{x-3}{x^2-9}$ has a infinite discontinuity at $x = -3$
a removable discontinuity at $x = 3$

39. $f(x) = \dfrac{|x+2|}{x+2}$ has a jump discontinuity at
$x = -2$ since $\lim_{x \to -2} f(x)$ does not exist.

50. $\lim_{x \to a} g(x) = \lim_{x \to a} \dfrac{x^2 - a^2}{x - a}$

$= \lim_{x \to a} (x + a) = 2a$

Find a such that $2a = 8 \implies a = 4$.

6. $g(x) = \dfrac{3}{2}x + 1$ is a line. Slope $= \dfrac{3}{2}$

8. Slope at $(2, 1) = \lim_{\Delta x \to 0} \dfrac{g(2 + \Delta x) - g(2)}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{5 - (2 + \Delta x)^2 - 1}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{5 - 4 - 4(\Delta x) - (\Delta x)^2 - 1}{\Delta x}$

$= \lim_{\Delta x \to 0} (-4 - \Delta x) = -4$

Home work

p 246 33,41

p 260 11, 15, 19, 23

$$|x+2| = \begin{cases} x+2 & x > -2 \\ 0 & x = -2 \\ -(x+2) & x < -2 \end{cases}$$

$\lim_{x \to -2^-} \dfrac{-(x+2)}{x+2} = -1$

$\lim_{x \to 2^+} \dfrac{x+2}{x+2} = 1$

9. Slope at $(0, 0) = \lim_{\Delta t \to 0} \dfrac{f(0 + \Delta t) - f(0)}{\Delta t}$

$= \lim_{\Delta t \to 0} \dfrac{3(\Delta t) - (\Delta t)^2 - 0}{\Delta t}$

$= \lim_{\Delta t \to 0} (3 - \Delta t) = 3$

Home work

p 247 36

p 260 21, 25a, 29a, 33

33. $f(x) = \dfrac{3x^2 - x - 2}{x - 1} = \dfrac{(3x + 2)(x - 1)}{x - 1}$

$\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} (3x + 2) = 5$

Removable discontinuity at $x = 1$.

Continuous on $(-\infty, 1) \cup (1, \infty)$.

41. $f(2) = 5$

Find c so that $\lim\limits_{x\to 2^+} (cx + 6) = 5$.

$c(2) + 6 = 5$

$2c = -1$

$c = -\frac{1}{2}$

11. $f(x) = 3$

$$f'(x) = \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{3 - 3}{\Delta x} = \lim_{\Delta x\to 0} \frac{0}{\Delta x} = \lim_{\Delta x\to 0} 0$$

$$= \lim_{\Delta x\to 0} 0 = 0$$

15. $h(s) = 3 + \frac{2}{3}s$

$$h'(s) = \lim_{\Delta s\to 0} \frac{h(s + \Delta s) - h(s)}{\Delta s}$$

$$= \lim_{\Delta s\to 0} \frac{3 + \frac{2}{3}(s + \Delta s) - \left(3 + \frac{2}{3}s\right)}{\Delta s}$$

$$= \lim_{\Delta s\to 0} \frac{\frac{2}{3}\Delta s}{\Delta s} = \frac{2}{3}$$

19. $f(x) = x^3 - 12x$

$$f'(x) = \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{[(x + \Delta x)^3 - 12(x + \Delta x)] - [x^3 - 12x]}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12x - 12\,\Delta x - x^3 + 12x}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12\,\Delta x}{\Delta x}$$

$$= \lim_{\Delta x\to 0} (3x^2 + 3x\,\Delta x + (\Delta x)^2 - 12) = 3x^2 - 12$$

23. $f(x) = \sqrt{x + 1}$

$$f'(x) = \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{\sqrt{x + \Delta x + 1} - \sqrt{x + 1}}{\Delta x} \cdot \left(\frac{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}}{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}}\right)$$

$$= \lim_{\Delta x\to 0} \frac{(x + \Delta x + 1) - (x + 1)}{\Delta x\left[\sqrt{x + \Delta x + 1} + \sqrt{x + 1}\right]}$$

$$= \lim_{\Delta x\to 0} \frac{1}{\sqrt{x + \Delta x + 1} + \sqrt{x + 1}}$$

$$= \frac{1}{\sqrt{x + 1} + \sqrt{x + 1}} = \frac{1}{2\sqrt{x + 1}}$$

41. $f(2) = 5$

Find c so that $\lim_{x \to 2^+} (cx + 6) = 5$.

$c(2) + 6 = 5$

$2c = -1$

$c = -\frac{1}{2}$

$$f(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$$

is $f(2)$ defined? yes

$f(2) = 5$

15. $h(s) = 3 + \frac{2}{3}s$

$$h'(s) = \lim_{\Delta s \to 0} \frac{h(s + \Delta s) - h(s)}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{3 + \frac{2}{3}(s + \Delta s) - \left(3 + \frac{2}{3}s\right)}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{\frac{2}{3}\Delta s}{\Delta s} = \frac{2}{3}$$

$$\frac{h(s+\Delta s) - h(s)}{\Delta s}$$

$$\frac{\cancel{3} + \cancel{\frac{2}{3}s} + \frac{2}{3}\Delta s - \cancel{3} - \cancel{\frac{2}{3}s}}{\Delta s}$$

$$\boxed{= \frac{2}{3}}$$

9. $f(x) = x^3 - 12x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[(x + \Delta x)^3 - 12(x + \Delta x)] - [x^3 - 12x]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\boxed{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3} - 12x - 12\,\Delta x - x^3 + 12x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2\,\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12\,\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (3x^2 + 3x\,\Delta x + (\Delta x)^2 - 12) = 3x^2 - 12$$

$(x+\Delta x)^3 = (x+\Delta x)(x+\Delta x)(x+\Delta x)$

$(x+\Delta x)(x^2+2x\Delta x+\Delta x^2)$