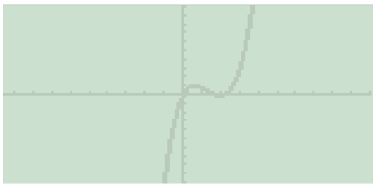
53. Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$.

This is not a unique solution.

For example, if $f(x) = (x + 1)^2$ and $g(x) = 2x$, then $(f \circ g)(x) = h(x)$ as well.



35. $f(t) = t^3 - 4t^2 + 4t$

$0 = t(t - 2)^2$

$t = 0, 2$

38. $f(x) = x^5 + x^3 - 6x$

$= x(x^4 + x^2 - 6)$

$= x(x^2 + 3)(x^2 - 2)$

$x = 0, \pm\sqrt{2}$

51. $f(x) = (x - 0)(x - (-2))(x - (-3))$

$= x(x + 2)(x + 3)$

$= x^3 + 5x^2 + 6x$

Note: $f(x) = ax(x + 2)(x + 3)$ has zeros $0, -2, -3$ for all real numbers $a \neq 0$.

57. $f(x) = (x - (-2))(x - (-2)) = (x + 2)^2 = x^2 + 4x + 4$

Note: If $a \neq 0$, then $f(x) = a(x^2 + 4x + 4)$ has degree 2 and zero $x = -2$, as does $f(x) = a(x + 2)(x - b)$ for any $b$ value.

zero $x: -2$ $n = 2$

$-2$ mult 2.

21.

| | | | | |
|---|---|---|---|---|
| 5 | 3 | −17 | 15 | −25 |
| | | 15 | −10 | 25 |
| | 3 | −2 | 5 | 0 |

$$\frac{3x^3 - 17x^2 + 15x - 25}{x - 5} = 3x^2 - 2x + 5 + 0$$

30.

| | | | | | | |
|---|---|---|---|---|---|---|
| −3 | 1 | −13 | 0 | 0 | −120 | 80 |
| | | −3 | 48 | −144 | 432 | −936 |
| | 1 | −16 | 48 | −144 | 312 | −856 |

$$\frac{x^5 - 13x^4 - 120x + 80}{x + 3} = x^4 - 16x^3 + 48x^2 - 144x + 312 - \frac{856}{x + 3}$$

39. $f(x) = x^3 - x^2 - 14x + 11$, $k = 4$

| | | | | |
|---|---|---|---|---|
| 4 | 1 | −1 | −14 | 11 |
| | | 4 | 12 | −8 |
| | 1 | 3 | −2 | 3 |

$f(x) = (x - 4)(x^2 + 3x - 2) + 3$

$f(4) = 4^3 - 4^2 - 14(4) + 11 = 3$

51.

| | | | | |
|---|---|---|---|---|
| 2 | 1 | 0 | −7 | 6 |
| | | 2 | 4 | −6 |
| | 1 | 2 | −3 | 0 |

$x^3 - 7x + 6 = (x - 2)(x^2 + 2x - 3)$

$= (x - 2)(x + 3)(x - 1)$

Zeros: $2, -3, 1$

Home work

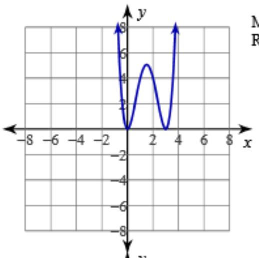
Complete

Handout

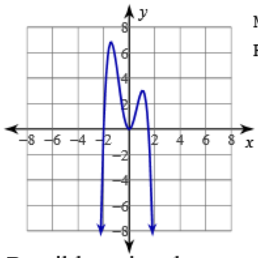
"Polynomial Functions"

Quiz Monday!

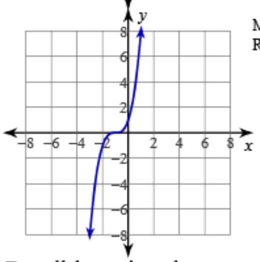
# Answers to Home work polynomial functions

1) 

Max # turns: 3
Real zeros: {0 mult. 2, 3 mult. 2}

2) 

Max # turns: 3
Real zeros: $\left\{0 \text{ mult. } 2, \frac{3}{2}, -2\right\}$

3) 

Max # turns: 2
Real zeros: {−1 mult. 3}

4) Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$
Factors to: $f(x) = (x-2)(x+3)(x-5)$
Zeros: $\{2, -3, 5\}$

5) Possible rational zeros: $\pm 1, \pm 2, \pm 4$
Factors to: $f(x) = (x-2)^2(x-1)$
Zeros: {2 mult. 2, 1}

6) Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$
Factors to: $f(x) = (x-2)(x+1)(x+3)$
Zeros: $\{2, -1, -3\}$

7) Possible rational zeros: $\pm 1, \pm 2, \pm 13, \pm 26$
Factors to: $f(x) = (x+13)(x+1)^2(x-2)$
Zeros: {−13, −1 mult. 2, 2}

Home work

Complete

"Zeros of Polynomial functions"

Quiz Monday!

# Answers to Homework zeros of polynomial functions

1) $f(x) = x^3 - 8x^2 + 15x$
2) $f(x) = x^3 - 3x^2 - 4x + 12$
3) $f(x) = 15x^3 + 47x^2 + 38x + 8$
4) $f(x) = x^4 - 3x^3 + 4x$
5) 18
6) $-15$
7) 20
8) 20
9) $0, \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$
10) $0, \pm 1, \pm 5, \pm 25, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{25}{2}$

Math 453 Name

$x^3 + \frac{1}{2}x^2 + 2x - \frac{7}{3}$

## Homework zeros of polynomial functions

Date

**Write a polynomial function of least degree with integral coefficients that has the given zeros.**

1) 0, 5, 3

2) 2, −2, 3

3) $-\frac{1}{3}, -\frac{4}{5}, -2$

4) 2 mult. 2, −1, 0

$3x^3+3x^2-3 = (x+2)(3x^2-3x+6)-15$

**Find the remainder when $f(x)$ is divided by $x - k$.**

$x-(-2)$

5) $f(x) = 8x^3 - 10x^2 - 8x + 10$
$k = 2$

6) $f(x) = 3x^3 + 3x^2 - 3$
$k = -2$

$(-2, -15)$

| | | | | |
|---|---|---|---|---|
| -2 | 3 | 3 | 0 | -3 |
| | | -6 | 6 | -12 |
| | 3 | -3 | 6 | -15 |

7) $f(x) = 5x^3 - 15x^2 - 15x$
$k = 4$

8) $f(x) = 4x^4 - 2x^3 + 16x^2 + 12x - 10$
$k = 1$

**State the possible rational zeros for each function.**

9) $f(x) = 3x^3 - 10x^2 + 8x + 0$

$f(x) = x(3x^2 - 10x + 8)$

0

p: ±1 ±2 ±4 ±8

q: ±1 ±3

List of possible rational zeros: 0, ±1, ±2, ±4, ±8, ±1/3, ±2/3, ±4/3, ±8/3

10) $f(x) = 2x^3 + 5x^2 - 25x$

# Answers to Homework zeros of polynomial functions

1) $f(x) = x^3 - 8x^2 + 15x$
2) $f(x) = x^3 - 3x^2 - 4x + 12$
3) $f(x) = 15x^3 + 47x^2 + 38x + 8$
4) $f(x) = x^4 - 3x^3 + 4x$
5) 18
6) $-15$
7) 20
8) 20
9) $0, \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$
10) $0, \pm 1, \pm 5, \pm 25, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{25}{2}$

Home work

p 165 41,43, 59, 65

p 182 7, 11, 15, 21, 25

Reminder Test Monday

4. 2 mult. 2, -1, 0

$f(x) = (x-2)^2 (x+1)(x)$

$(x^2 - 2x - 2x + 4)(x^2 + x)$

$(x^2 - 4x + 4)(x^2 + x)$

$x^4 - 4x^3 + 4x^2 + x^3 - 4x^2 + 4x$

$f(x) = x^4 - 3x^3 + 4x$

Oct 25-9:09 AM

**41.** $f(x) = 15x^4 + 10x^3 - 6x^2 + 14, \; k = -\frac{2}{3}$

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 15 & 10 & -6 & 0 & 14 \\ & & -10 & 0 & 4 & -\frac{8}{3} \\ \hline & 15 & 0 & -6 & 4 & \frac{34}{3} \end{array}$$

$f(x) = \left(x + \frac{2}{3}\right)(15x^3 - 6x + 4) + \frac{34}{3}$

$f\left(-\frac{2}{3}\right) = 15\left(-\frac{2}{3}\right)^4 + 10\left(-\frac{2}{3}\right)^3 - 6\left(-\frac{2}{3}\right)^2 + 14 = \frac{34}{3}$

**43.** $f(x) = x^3 + 3x^2 - 2x - 14, \; k = \sqrt{2}$

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2 + 3\sqrt{2} & 6 \\ \hline & 1 & 3 + \sqrt{2} & 3\sqrt{2} & -8 \end{array}$$

$f(x) = \left(x - \sqrt{2}\right)\left[x^2 + \left(3 + \sqrt{2}\right)x + 3\sqrt{2}\right] - 8$

$f\left(\sqrt{2}\right) = \left(\sqrt{2}\right)^3 + 3\left(\sqrt{2}\right)^2 - 2\sqrt{2} - 14 = -8$

**59.** $f(x) = 2x^3 + x^2 - 5x + 2;$

Factors: $(x + 2), (x - 1)$

(a)
$$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 & 2 & -3 & 1 \\ & & 2 & -1 \\ \hline & 2 & -1 & 0 \end{array}$$

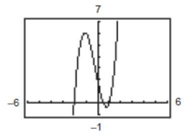
Both are factors of $f(x)$ since the remainders are zero.

(b) The remaining factor of $f(x)$ is $(2x - 1)$.

(c) $f(x) = (2x - 1)(x + 2)(x - 1)$

(d) Zeros: $\frac{1}{2}, -2, 1$

(e)




**65.** $f(x) = 2x^3 - x^2 - 10x + 5;$

Factors: $(2x - 1), \left(x + \sqrt{5}\right)$

(a)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -10 & 5 \\ & & 1 & 0 & -5 \\ \hline & 2 & 0 & -10 & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{5} & 2 & 0 & -10 \\ & & -2\sqrt{5} & 10 \\ \hline & 2 & -2\sqrt{5} & 0 \end{array}$$

Both are factors since the remainders are zero.

(b) $2x - 2\sqrt{5} = 2\left(x - \sqrt{5}\right)$

This shows that $\dfrac{f(x)}{\left(x - \frac{1}{2}\right)\left(x + \sqrt{5}\right)} = 2\left(x - \sqrt{5}\right),$

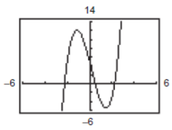
so $\dfrac{f(x)}{(2x - 1)\left(x + \sqrt{5}\right)} = x - \sqrt{5}.$

The remaining factor is $\left(x - \sqrt{5}\right)$.

(c) $f(x) = \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)(2x - 1)$

(d) Zeros: $-\sqrt{5}, \sqrt{5}, \frac{1}{2}$

(e)




**7.** $f(x) = x^3 + 3x^2 - x - 3$

Possible rational zeros: $\pm 1, \pm 3$

Zeros shown on graph: $-3, -1, 1$

**11.** $f(x) = x^3 - 6x^2 + 11x - 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6) = (x - 1)(x - 2)(x - 3)$

Thus, the real zeros are 1, 2, and 3.

**15.** $h(t) = t^3 + 12t^2 + 21t + 10$

Possible rational zeros: $\pm 1, \pm 2, \pm 5, \pm 10$

$$\begin{array}{r|rrrr} -1 & 1 & 12 & 21 & 10 \\ & & -1 & -11 & -10 \\ \hline & 1 & 11 & 10 & 0 \end{array}$$

$$\begin{aligned} t^3 + 12t^2 + 21t + 10 &= (t + 1)(t^2 + 11t + 10) \\ &= (t + 1)(t + 1)(t + 10) \\ &= (t + 1)^2(t + 10) \end{aligned}$$

Thus, the zeros are $-1$ and $-10$.

**21.** $z^4 - z^3 - 2z - 4 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 2 & -4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

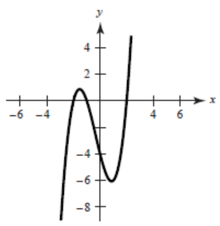
$z^4 - z^3 - 2z - 4 = (z + 1)(z - 2)(z^2 + 2)$

The only real zeros are $-1$ and 2.

**25.** $f(x) = x^3 + x^2 - 4x - 4$

(a) Possible rational zeros: $\pm 1, \pm 2, \pm 4$

(b)




(c) The zeros are: $-2, -1, 2$.

Oct 26-8:20 AM