Do the following questions for practice

with Rolles, MVT, and IVT

p 236 73, 76

p 326 10, 15, 16

p 327 27, 28, 30, 32

73. $f(x) = x^2 + x - 1$

f is continuous on $[0, 5]$.

$f(0) = -1$ and $f(5) = 29$.

$-1 < 11 < 29$

The Intermediate Value Theorem applies.

$$x^2 + x - 1 = 11$$
$$x^2 + x - 12 = 0$$
$$(x + 4)(x - 3) = 0$$
$$x = -4 \text{ or } x = 3$$

$c = 3$ ($x = -4$ is not in the interval.)

Thus, $f(3) = 11$.

10. $f(x) = x^2 - 5x + 4$, $[1, 4]$

$f(1) = f(4) = 0$

f is continuous on $[1, 4]$. f is differentiable on $(1, 4)$.

Rolle's Theorem applies.

$$f'(x) = 2x - 5$$
$$2x - 5 = 0 \Rightarrow x = \frac{5}{2}$$

c value: $\frac{5}{2}$

15. $f(x) = \dfrac{x^2 - 2x - 3}{x + 2}$, $[-1, 3]$

$f(-1) = f(3) = 0$

f is continuous on $[-1, 3]$. (**Note:** The discontinuity, $x = -2$, is not in the interval.) f is differentiable on $(-1, 3)$. Rolle's Theorem applies.

$$f'(x) = \frac{(x + 2)(2x - 2) - (x^2 - 2x - 3)(1)}{(x + 2)^2} = 0$$
$$\frac{x^2 + 4x - 1}{(x + 2)^2} = 0$$
$$x = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$$

c value: $-2+\sqrt{5}$

16. $f(x) = \dfrac{x^2 - 1}{x}$, $[-1, 1]$

$f(-1) = f(1) = 0$

f is not continuous on $[-1, 1]$ since $f(0)$ does not exist.
Rolle's Theorem does not apply.

76. $f(x) = \dfrac{x^2 + x}{x - 1}$

f is continuous on $\left[\frac{5}{2}, 4\right]$. The nonremovable discontinuity, $x = 1$, lies outside the interval.

$f\left(\frac{5}{2}\right) = \frac{35}{6}$ and $f(4) = \frac{20}{3}$.

$\frac{35}{6} < 6 < \frac{20}{3}$

The Intermediate Value Theorem applies.

$$\frac{x^2 + x}{x - 1} = 6$$
$$x^2 + x = 6x - 6$$
$$x^2 - 5x + 6 = 0$$
$$(x - 2)(x - 3) = 0$$
$$x = 2 \text{ or } x = 3$$

$c = 3$ ($x = 2$ is not in the interval.)

Thus, $f(3) = 6$.

27. $f(x) = x^2$ is continuous on $[-2, 1]$ and differentiable on $(-2, 1)$.

$$\frac{f(1) - f(-2)}{1 - (-2)} = \frac{1 - 4}{3} = -1$$

$f'(x) = 2x = -1$ when $x = -\frac{1}{2}$. Therefore,

$$c = -\frac{1}{2}.$$

28. $f(x) = x(x^2 - x - 2)$ is continuous on $[-1, 1]$ and differentiable on $(-1, 1)$.

$$\frac{f(1) - f(-1)}{1 - (-1)} = -1$$

$$f'(x) = 3x^2 - 2x - 2 = -1$$

$$(3x + 1)(x - 1) = 0$$

$$c = -\frac{1}{3}$$

30. $f(x) = (x + 1)/x$ is continuous on $[1/2, 2]$ and differentiable on $(1/2, 2)$.

$$\frac{f(2) - f(1/2)}{2 - (1/2)} = \frac{(3/2) - 3}{3/2} = -1$$

$$f'(x) = \frac{-1}{x^2} = -1$$

$$x^2 = 1$$

$$c = 1$$

32. $f(x) = x^3$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$.

$$\frac{f(1) - f(0)}{1 - 0} = \frac{1 - 0}{1} = 1$$

$$f'(x) = 3x^2 = 1$$

$$x = \pm\frac{\sqrt{3}}{3}$$

In the interval $(0, 1)$: $c = \frac{\sqrt{3}}{3}$.