Confidence Intervals and Tests of Significance

1. An automobile manufacturer believes that the mean mileage per gallon of one of its new models exceeds the mean EPA rating of 43 miles per gallon. To gain evidence to support its belief, the manufacturer randomly selected forty of the cars and recorded the miles per gallon for each over a 100-mile course. The mean and standard deviation of the mileages per gallon for the sample of forty cars was 43.6 and 1.3 miles per gallon respectively.
   a. Construct a 90% confidence interval for the mileage.
   b. Do the data provide sufficient evidence to support the manufacturer’s belief?

2. Small increases in the mean level of bills for monthly cell phone bills produce substantial increases or decreases in the profits for telephone companies. A telephone company’s records indicate that the amounts paid by private customers per month for their cell phone have a distribution with mean $34.60 and standard deviation of $21.21.
   a. If a sample of 20 customers were found to have an average bill of $42, construct a 95% confidence interval.
   b. Does part “a” lead you to believe that the company might be overcharging? Explain.
   c. Suppose the margin of error wants to be reduced to ± $5. How many customers will need to be sampled?

3. Bookstores like towns with a good percentage of college educated people. National data shows that 71% of college graduates have read a book in the past year, compared to 54% of the general population age 18 and over. A bookstore is doing a market survey in a certain county to decide whether to open a bookstore. The company likes to open bookstores if a population is above the national educational level. The bookstore takes a simple random sample of 1,000 people age 18 and over and finds the average educational level to be 14 years with a standard deviation of 5 years.
   a. Construct a 90% confidence interval for the mean educational level in the county.
   b. The nationwide average educational level to be 13 years of schooling completed person age 18 and over. Do the data provide sufficient evidence to conclude that a new bookstore should be built?

4. A pain reliever currently being used in a hospital is known to bring relief to patients in a mean time of 3.5 minutes. To compare a new pain reliever with the one currently being used, the new drug is administered to a random sample of fifty patients. The mean time to relief for the sample of patients is 2.8 minutes and the standard deviation is 1.14 minutes.
   a. Construct a 98% confidence interval for the mean time till relief.
   b. Do the data provide sufficient evidence to conclude that the new drug was effective in reducing the mean time until a patient receives relief from pain?

5. In a study of air-bag effectiveness, it was found that in 821 crashes of midsize cars equipped with air bags, 46 of the crashes resulted in hospitalization of the drivers.
   a. Give a 95% confidence interval for the percent of crashes resulting in hospitalization. Interpret the confidence interval and the confidence level.
   b. Test the claim that the air-bag hospitalization rate is lower than the 7.8% rate for crashes of mid-size cars equipped with automatic safety belts.
   c. How large a sample would be needed to obtain the same margin of error in part “a” for a 99% confidence interval?
6. A new insect spray is to be compared with a spray that is currently in use. Two rooms of equal size are sprayed with the same amount of the designated spray. Two hundred fifty insects are released into each room and after 1 hour, the numbers of dead insects are counted. The new insect spray kill 60% of the insects while of current spray killed 45%.

   a. Find a 98% confidence interval for the difference in the rates of kill for the two sprays. Interpret the interval.

   b. Do the data provide sufficient evidence to indicate that the new spray is more effective than the current spray in controlling the insects?

7. Based on information from Consumer Reports, a random sample of 86 thirty-gram servings of Duncan Hines Chocolate Chip Cookies had a sample mean of 132 calories with sample standard deviation of 27 calories. A random sample of 75 Pepperidge Farm Chocolate Chip Cookies had a sample mean of 124 calories with sample standard deviation of 33 calories.

   a. Construct a 96% confidence interval for the mean difference in calories?

   b. Does this information indicate that there is a difference in the average number of calories for these two cookies brands?

8. Given Ho: μ ≤ 12, for which confidence interval should you reject H0?
   A) (13, 16)
   B) (10, 13)
   C) (11.5, 12.5)

9. Given Ho: p ≥ 0.45, for which confidence interval should you reject H0?
   A) (0.40, 0.50)
   B) (0.32, 0.40)
   C) (0.42, 0.47)

10. Given Ha: μ < 23.5 and α = 0.10, which level of confidence should you use to test the claim?
    A) 99%
    B) 95%
    C) 80%
    D) 90%

11. Given Ha: μ < 23.5 and α = 0.05, which level of confidence should you use to test the claim?
    A) 80%
    B) 90%
    C) 95%
    D) 99%

12. The test statistic in a right-tailed test is z = 0.52 for α = 0.05
    A) fail to reject the null hypothesis
    B) reject the null hypothesis

13. The test statistic in a right-tailed test is z = 1.43 for α = 0.10
    A) fail to reject the null hypothesis
    B) reject the null hypothesis

14. The test statistic in a left-tailed test is z = -1.83 for α = 0.02
    A) fail to reject the null hypothesis
    B) reject the null hypothesis

15. The test statistic in a left-tailed test is t = -2.05 for a critical value of -2.576
    A) reject the null hypothesis
    B) fail to reject the null hypothesis

16. The test statistic in a two-tailed test is z = 1.95 for a critical value of ±1.96
    A) reject the null hypothesis
    B) fail to reject the null hypothesis

State whether the conclusion is reject or fail to reject the null hypothesis

17. The test statistic is z = 2.1 for:
    a. right tail with α = 0.05
    b. right tail with α = 0.01
    c. two tail with α = 0.05
Answers

1. a. (43.254, 43.946)
   b. Since 43 mpg is not in our interval and Ho: $\mu = 43$ and Ha: $\mu > 43$, we will reject the population mean EPA rating of 43 mpg at the 5% significance level.

2. a. (32.09, 51.91)
   b. Since $42 is in our interval and Ho: $\mu = 42$ and Ha: $\mu \neq 42$, we will fail to reject the population mean cell phone bill is $42 at the 5% significance level.
   c. 69.128 so 70

3. a. (13.74, 14.26)
   b. Since 13 is not in our interval and Ho: $\mu = 13$ and Ha: $\mu > 13$, we will reject the population mean educational level is 13 at the 5% significance level.

4. a. (2.4123, 3.1877)
   b. Since 3.5 is not in our interval and Ho: $\mu = 13$ and Ha: $\mu < 3.5$, we will reject the population mean time to receive relief from pain is 3.5 minutes at the 1% significance level.

5. a. (0.0403, 0.07176)
   b. Since 7.8% is not in our interval and Ho: $p = 0.078$ and Ha: $p < 0.078$, we will reject the population proportion air bag hospitalization rate is 7.8% at the 2.5% significance level.
   c. 1418.15 so 1419

6. a. (0.0473, 0.2527)
   b. Since 0 is not in our interval and Ho: $p1 – p2 = 0$ and Ha: $p1 – p2 > 0$, we will reject the population proportion that there is no difference in number of insects killed using the new spray versus the old spray at the 1% significance level.

7. a. (-2.025, 18.025)
   b. Since 0 is in our interval and Ho: $\mu1 - \mu2 = 0$ and Ha: $\mu1 - \mu2 \neq 0.078$, we will reject the population proportion air bag hospitalization rate is 7.8% at the 2.5% significance level.

8. A
9. B
10. D
11. B
12. A
13. B
14. A
15. B
16. B
17. a. reject the null hypothesis
   b. fail to reject the null hypothesis
   c. reject the null hypothesis