Directions: You may use a scientific calculator, but you MUST SHOW ALL WORK TO RECEIVE full credit. Have a great February break!! If necessary, round to the nearest hundredth.

1. Solve for $x$ in simplest radical form.
   $\log_4 x + \log_4 (x - 2) = 2$
   
   $\log_4 x(x - 2) = 2$
   
   $x^2 - 2x = 16$
   
   $x^2 - 2x - 16 = 0$
   
   $x = \frac{2 \pm \sqrt{4 - 4(1)(-16)}}{2}$
   
   $x = \frac{2 \pm \sqrt{68}}{2}$
   
   $x = 1 + \sqrt{17}$

2. Solve for $x$.
   
   $\left(\frac{1}{125}\right)^{2x+1} = 25^{3x}$
   
   $(5^{-3})^{2x+1} = (5^2)^{3x}$
   
   $5^{-6x-3} = 5^{6x}$
   
   $-6x - 3 = 6x$
   
   $x = -\frac{3}{12}$

3. Consider the functions given by $p(x) = 5^{3x}$. Express $f(h(p(x)))$ in simplest form.
   
   $f(x) = e^x$
   
   $h(x) = \ln x$
   
   $e \ln 5^{3x} = 5^{3x}$

4. Sketch a graph of the function given by $y = \log_3 (|x| - 3) - 2$. Include and label and asymptotes and/or key points on your graph. Please clearly indicate your transformations or support your answer in some manner.

   $y = \log_2 x \rightarrow y = \log_2 (x - 3) \rightarrow$
   
   $y = \log_2 (|x| - 3) \rightarrow y = \log_2 (|x| - 3) - 2$
5. Solve for $x$. (Exact answer in simplest form)

\[
\left(\frac{1}{\sqrt{2}}\right)^{\frac{3}{4}} = \pm 2^{\frac{3}{4}}
\]

\[
= \pm \sqrt[4]{2^3}
\]

6. Solve for $x$. (Exact answer in simplest form)

\[
\frac{15}{1 + e^{-2x+1}} = 4
\]

\[
15 = 4(1 + e^{-2x+1})
\]

\[
15 = 4 + 4e^{-2x+1}
\]

\[
11 = 4e^{-2x+1}
\]

\[
\frac{11}{4} = e^{-2x+1}
\]

\[
\ln \left(\frac{11}{4}\right) = -2x+1
\]

\[
\ln \left(\frac{11}{4}\right) = -2x+1 \rightarrow -\frac{\ln \left(\frac{11}{4}\right) - 1}{-2} = x
\]

7. If $x = \log_5 a$ and $y = \log_5 c$, express the following in terms of $x$ and $y$.

\[
\log_5 \left(\frac{.04a^3}{c^2}\right)
\]

\[
= \log_5 (100a^3) + \log_5 a^3 - \log_5 c^2
\]

\[
= \log_5 \left(\frac{1}{5}\right)
\]

\[
= \log_5 5^{-2}
\]

\[-2x + 3x - 2y
\]

8. Solve for $x$. $\log_3 \left(\log_5 \left(\log_3 x\right)\right) = -1$

(Exact answer in simplest form)

\[
3^{-1} = \log_5 \left(\log_3 x\right)
\]

\[
\frac{1}{3} = \log_5 \left(\log_3 x\right)
\]

\[
8^{\frac{1}{3}} = \log_5 x
\]

\[
2 = \log_5 x
\]

\[
\log_5 x = \frac{5^2}{5} = 25
\]

9. The graph below represents a function. Write a possible equation for this function.

\[
y = \log x
\]

\[
y = -\log x
\]

\[
y = \log \left(\frac{1}{x}\right)
\]

\[
y = \log_a (x+2) + k
\]

10. Solve for $x$. $6 - \log_5 (3x - 2) = 4$

\[
-\log_5 (3x - 2) = -2
\]

\[
\log_5 (3x - 2) = 2
\]

\[
25 = 3x - 2
\]

\[
x = 9
\]

11. Solve for $x$. $\log x + \log (x-1) = \log (x)$

\[
\log \left(x^2 - x\right) = \log (x)
\]

\[
x^2 - x = x
\]

\[
x^2 - 2x = 0
\]

\[
X(Y - 2) = 0
\]

\[
x > 0, 0 \leq Y = 2
\]
12. Solve for \( x \). \( 15 \log_3 x + 2 = \log_3 x \)

(Exact answer)

\[
\frac{15}{\log_3 x} + 2 = 1 \Rightarrow \frac{15}{\log_3 x} = -2 \Rightarrow 15 = -2 \log_3 x \Rightarrow 15 = -2x \Rightarrow x = \frac{15}{-2} \Rightarrow x = -\frac{15}{2}
\]

13. Solve for \( x \). \( e^{2x} - 4e^x = -3 \). Please provide an exact answer.

\[
\left( e^x \right)^2 - 4e^x + 3 = 0
\]

\[
\begin{align*}
\left( e^x - 1 \right) \left( e^x - 3 \right) &= 0 \\
e^x &= 1 \quad e^x = 3 \\
x &= 0 \quad x = \ln 3
\end{align*}
\]

14. Consider the function given by \( f(x) = \left( \frac{2}{3} \right)^x + 1 \). Find \( f^{-1}(x) \) and state the domain and range of \( f^{-1}(x) \).

\[
\begin{align*}
x &= \left( \frac{2}{3} \right)^y + 1 \\
\log_{\frac{2}{3}} (x-1) &= y \\
D: x > 1 & \quad (1, \infty) \\
R: & \quad (-\infty, \infty)
\end{align*}
\]

15. You are observing a mystery radioactive isotope. At 4 pm, there are 2.5 grams and at 9 pm, there are 1.7 grams.

a. What's the half-life (please round to the nearest thousandth of an hour)?

\[
\begin{align*}
1.7 &= 2.5 \left( \frac{1}{2} \right)^t \\
\ln \left( \frac{1.7}{2.5} \right) &= t \cdot \ln \left( \frac{1}{2} \right) \\
t &= \frac{\ln \left( \frac{1.7}{2.5} \right)}{\ln \left( \frac{1}{2} \right)} \\
x &= \frac{5}{\ln \left( \frac{1}{2} \right)} \\
&= 8.986
\end{align*}
\]

b. How long will it take for 15% of the mysterious isotope to decay? (please round to the nearest hundredth of an hour.)

\[
\begin{align*}
.85 &= 1 \left( .5 \right)^t \\
\ln .85 &= t \cdot \ln .5 \\
2.11 &= t \cdot \ln .5 \\
\end{align*}
\]

16. Suppose you invest $2017 in a bank that pays 5.25% annual interest. Find to the nearest cent, the amount of money in the account after 15 years if the interest is compounded:

a. Monthly

\[
A = 2017 \left( 1 + \frac{.0525}{12} \right)^{12 \cdot 15} \Rightarrow A = 4425.55
\]

b. Continuously

\[
A = 2017 e^{.0525 \cdot 15} \Rightarrow A = 4433.15
\]